Multiparty Computation

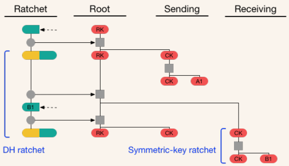
Hari

Cryptography

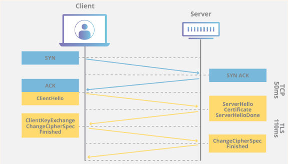
Cryptography

Secure Communication

- AES, RSA, etc.



Double Ratchet: Signal



TLS: Cloudflare

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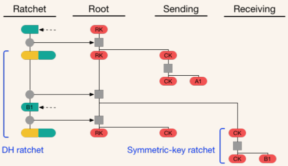

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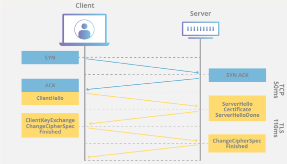
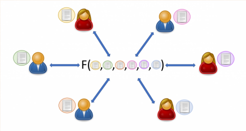
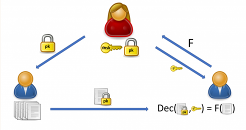


TLS: Cloudflare

Secure **Computation**

- Modern day constructions
- MPC, FE, FHE, and more
- Secure Voting and Auctions, Cryptocurrency, Secure ML





Secure Computation: COSIC

Secure Multiparty Computation

- Can we compute a function of data from multiple parties **securely**?

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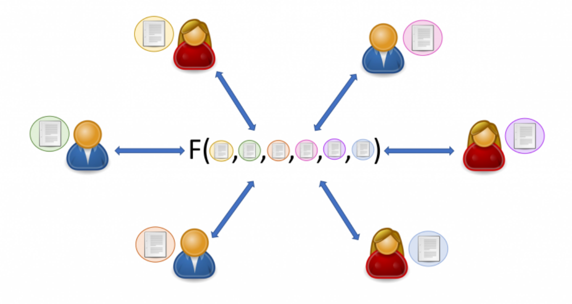

Secure MPC: Cosic

Secure Multiparty Computation

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- Train a machine learning model without revealing training datasets
- Hiding auction bids on a smart contract

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- Generalized to multiple parties by Goldreich, Micali, and Widgerson

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GMW Protocol

Introduction to GMW

Given a function $\mathcal{F}(x_1, x_2, \ldots x_n)$ representable as a Boolean circuit $\mathcal{C}$, how do we evaluate it?

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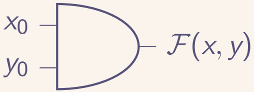

Example circuit

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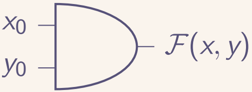
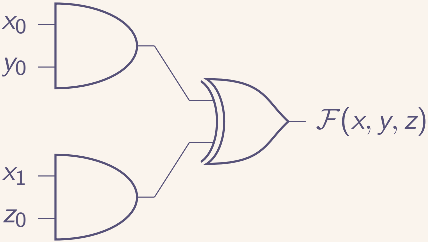


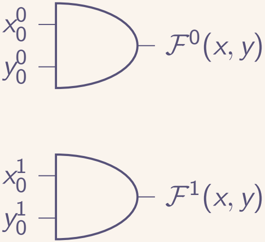
Example circuit



Slightly more complicated circuit

Introduction to GMW

- Idea: Each party gives a "piece" of their data to the other



Example split circuit (Two parties)

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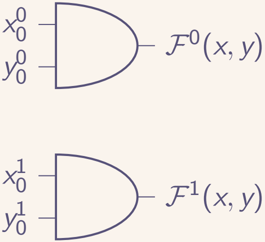

Example split circuit (Two parties)

- With these input pieces, each party can evaluate the circuit
- Known as **secret sharing**

Secret Sharing

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Secret Sharing

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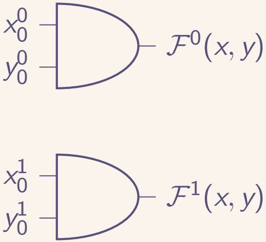
- Choose random bits $r_0, r_1, r_2, \ldots, r_{n-1}$
- Set $r_n = a \oplus r_0 \oplus r_1 \oplus \ldots \oplus r_{n-1}$
- Provide each party with one of the r_i
- Notice that all parties must get together to find a
- The r_i are known as **secret shares** or **shares**

GMW



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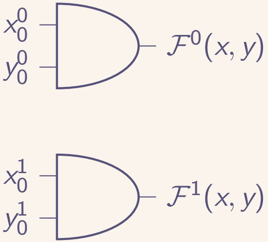
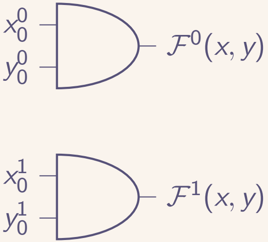


Example split circuit (Two parties)

- Party x creates the shares x_0^0, x_0^1 of bit x_0
- Party y creates the shares y_0^0, y_0^1 of bit y_0

GMW



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- Party x creates the shares x_0^0, x_0^1 of bit x_0
- Party y creates the shares y_0^0, y_0^1 of bit y_0
- Party x gets x_0^0, y_0^0, Party y gets x_0^1, x_0^1

How do we evaluate this "shared" circuit?

Evaluation

- Firstly, we assume our circuit only has AND, NOT, and XOR gates (which is universal)

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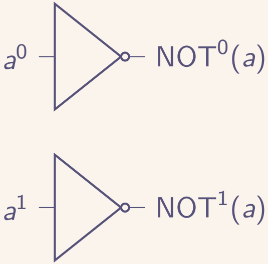
- When we evaluate a gate, **we want to get a secret share of the gate output**
- This allows parties to continue evaluating the next gate

Evaluation: NOT

$$a^0 \oplus a^1 = a$$



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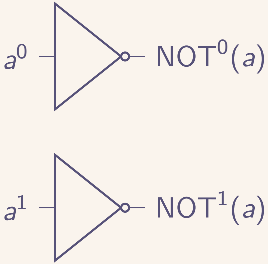

Shared NOT Circuit

- Notice that if we set a^0 to $\text{NOT}(a^0)$, we get

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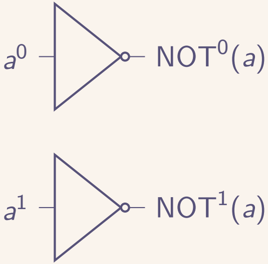

Shared NOT Circuit

- Notice that if we set a^0 to NOT(a^0), we get

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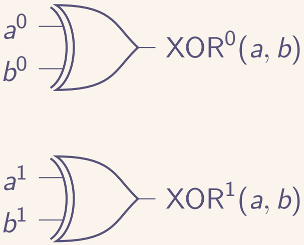
- One party just has to flip their share

Evaluation: XOR

$$a^0 \oplus a^1 = a,\ b^0 \oplus b^1 = b$$



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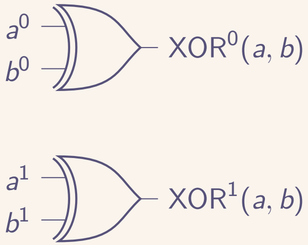

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- Notice that $(a^0 \oplus b^0) \oplus (a^1 \oplus b^1) = a \oplus b$

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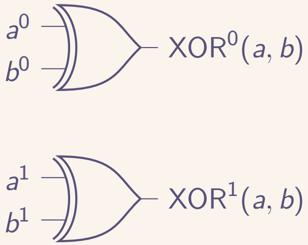
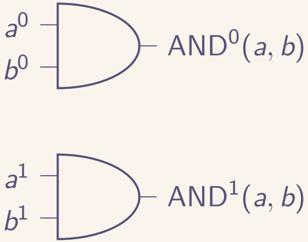


Shared XOR Circuit

- Notice that $(a^0 \oplus b^0) \oplus (a^1 \oplus b^1) = a \oplus b$
- Then we can have each party evaluate $\mathrm{XOR}^i(a, b) = a^i \oplus b^i$

Evaluation: AND

$$a^0 \oplus a^1 = a,\ b^0 \oplus b^1 = b$$



Shared AND Circuit

How do we do this?

Detour: Oblivious Transfer

Oblivious Transfer

Definition (One out of n Oblivious Transfer)

We have two parties, the sender $\mathcal{S}$ and receiver $\mathcal{R}$.
$\mathcal{S}$ has n secrets $x_0, x_1, \ldots x_n$.
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$\mathcal{R}$ has a selection value v from 1 to n.
An **Oblivious Transfer** protocol is a protocol where

- $\mathcal{R}$ receives x_v without learning any of the other secrets
- $\mathcal{S}$ does not learn v

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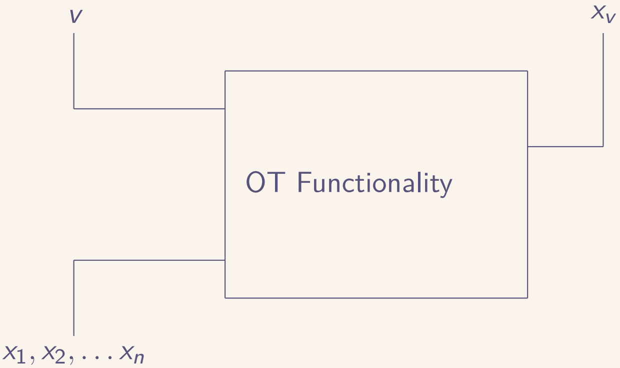
- (ask me after for an OT example)

Oblivious Transfer



Oblivious Transfer

Back to GMW!

Evaluation: AND

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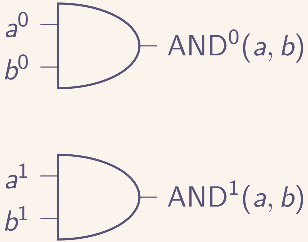

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- The first party can select a random bit $r \in \{0, 1\}$, and create 4 "possible" secret shares

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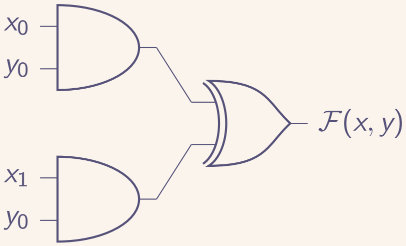
- Then the first party uses 1 out of 4 Oblivious Transfer to send the share
- Notice that we get a secret share of the AND value!

GMW

- And we're done! We can evaluate any circuit, and then at the end we reconstruct the secret with the shares.



Example Circuit

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- NOT gates: Only one person still needs to flip their share

$$\neg(a^0 \oplus a^1 \oplus a^2 \dots) = (\neg a^0) \oplus a^1 \oplus a^2 \dots$$

- XOR gates: All parties still xor their shares together

$$(a^0 \oplus a^1 \oplus a^2 \dots) \oplus (b^0 \oplus b^1 \oplus b^2 \dots)$$
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- AND gates: ???

GMW: Multiple Parties

- AND gates: We can see

$$
\begin{aligned}
&(a^0 \oplus a^1 \oplus a^2 \dots) \wedge (b^0 \oplus b^1 \oplus b^2 \dots) \\
&= \left(\bigoplus_{i \in [n]} a^i \wedge b^i \right) \oplus \left(\bigoplus_{i \neq j} a^i \wedge b^j \right)
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- **Reduce the multiparty problem into a set of two party problems**

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- In the end they can reconstruct the output by using output shares

More Fun Things

- This is just the surface: many more questions
- Can you evaluate functions securely if there are malicious parties?
- Can we improve round complexity and communication efficiency?

More Fun Things

- Very useful in the world of secure computation
- For example, say the circuit is a circuit which takes in an ML classifier and an image, and outputs the classification
- Allows doctors to provide medical images / data to ML models without violating HIPAA

More Fun Things

Bonus: Funny card trick (related to MPC)!!