Security Estimation for Post-Quantum Cryptosystems

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January 26, 2023

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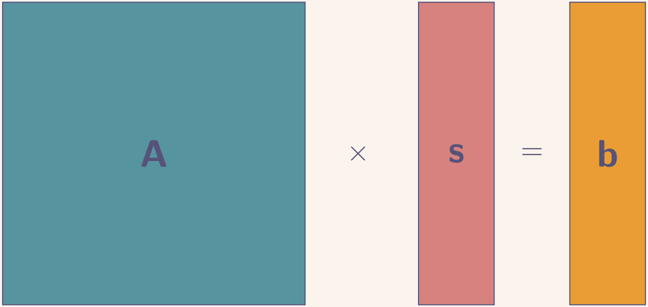
- Learning with Errors problem (**LWE**), lattice problems seem hard for classical and quantum computers
- **Motivating question**: Can we crack* LWE? (side channel security estimation)

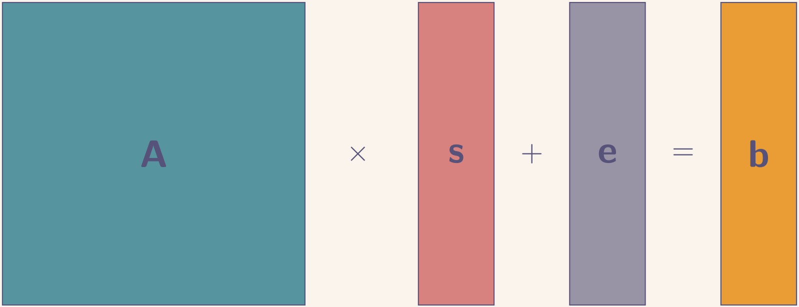
LWE (Almost)

Given $\mathbf{A} \in \mathbb{Z}_q^{m \times n}$ and $\mathbf{b} \in \mathbb{Z}_q^m$, find $\mathbf{s}$ where $\mathbf{As} = \mathbf{b} \mod q$

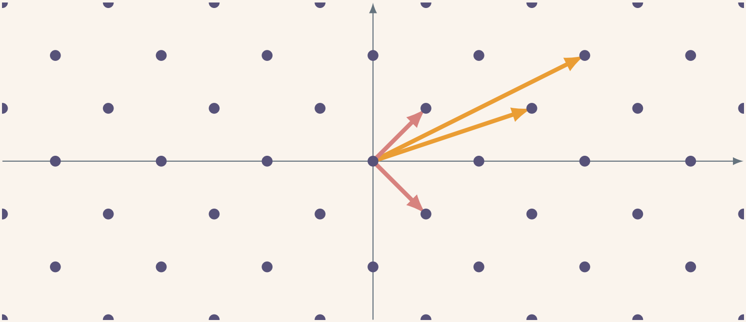


LWE

Now given an *unknown*, small Gaussian error **e**, find **s**



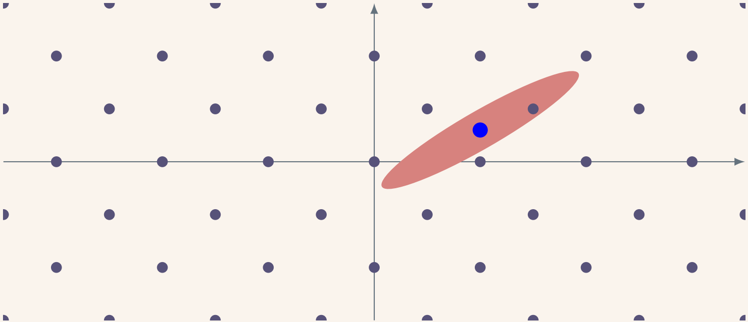
Lattices



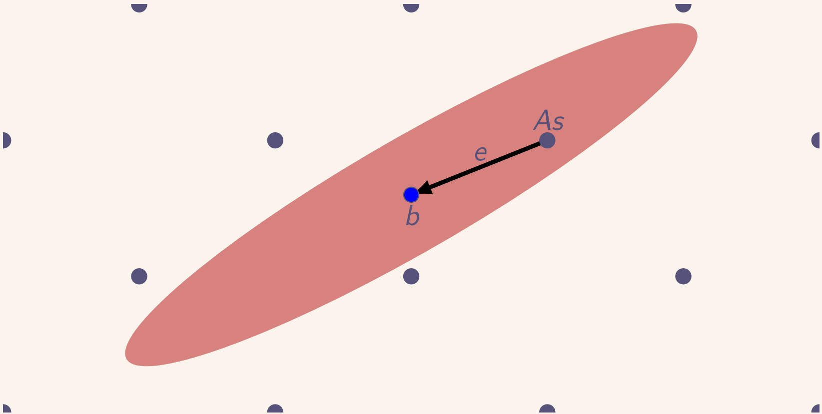
$$\mathcal{L}(B) \stackrel{def}{=} \{Bx \mid x \in \mathbb{Z}^n\}$$

Ellipsoidal Bounded Distance Decoding (EBDD)

Given a lattice Λ, an ellipsoid E with center μ and shape Σ, and the promise that there exists a unique lattice point $x \in \Lambda \cap E$, find x



The EBDD Embedding (LWE $\rightarrow$ EBDD)



LWE: $As + e = b$

Shape: $||As - b||_2^2 \leq m \cdot \sigma^2$ (big simplification)

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- BKZ is our exponential-time algorithm for SVP (its a pretty good algorithm)

EBDD Reduces to SVP

We have techniques to turn EBDD into the SVP!

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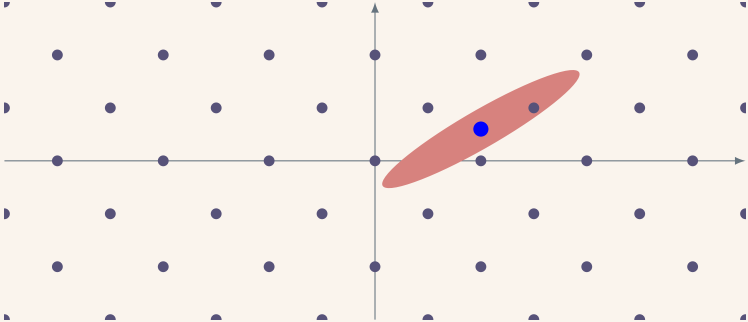

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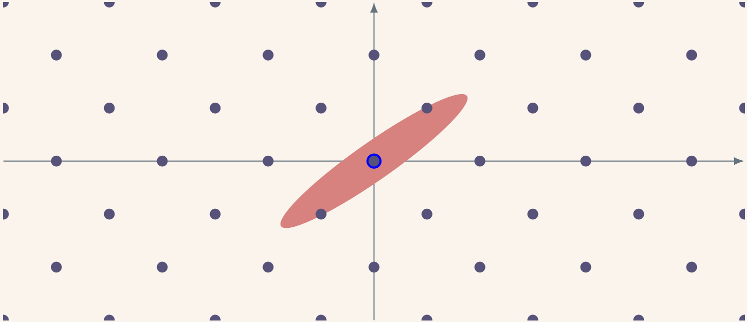

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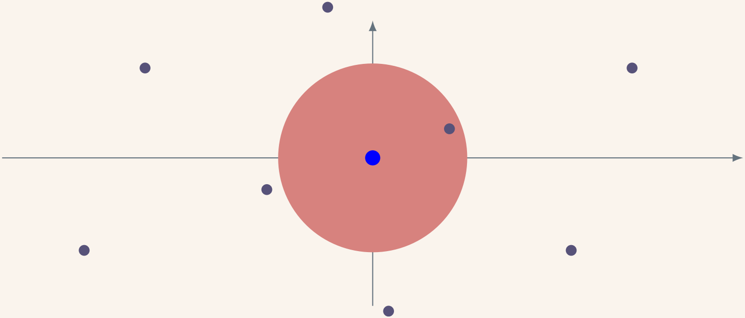
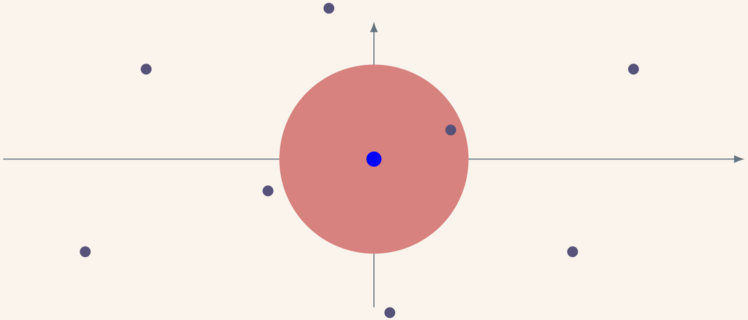


BKZ!

EBDD Reduces to SVP



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smaller EBDD ellipsoids $\Longrightarrow$ **easier** SVP instances

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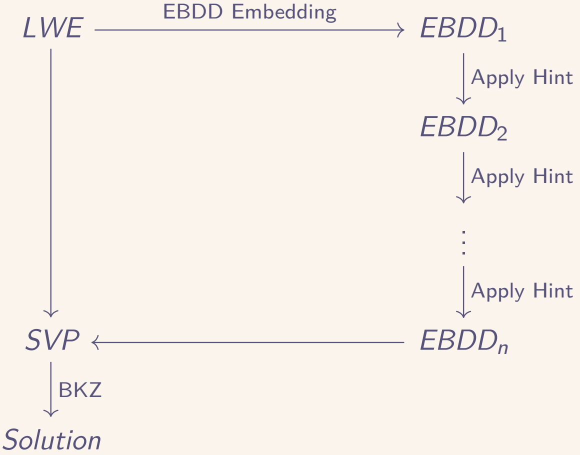
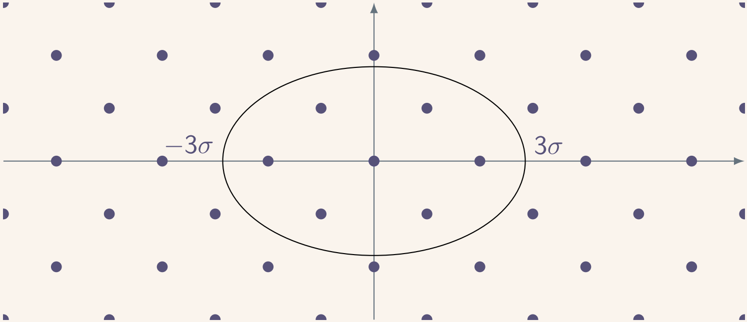
- Apply hints, side-channel information to decrease ellipsoid volume

Background: Recap



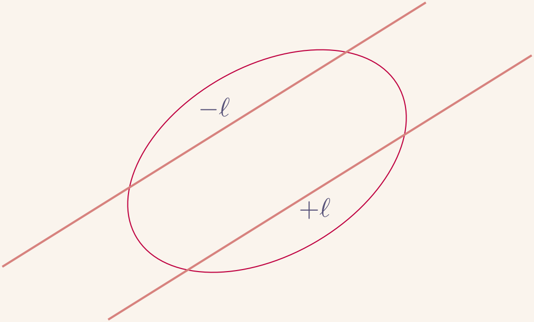
Ellipsoid Based on Distributional Knowledge

$\mathbf{s}, \mathbf{e}$ are Gaussian-distributed, so 95% confidence intervals can define a new ellipsoid



Alpha Cuts

- The error term **e** is small: in particular, each e_i is between $-\ell$ and $+\ell$.
- This gives *m* parallel lines intersecting our ellipsoid, with the secret in between



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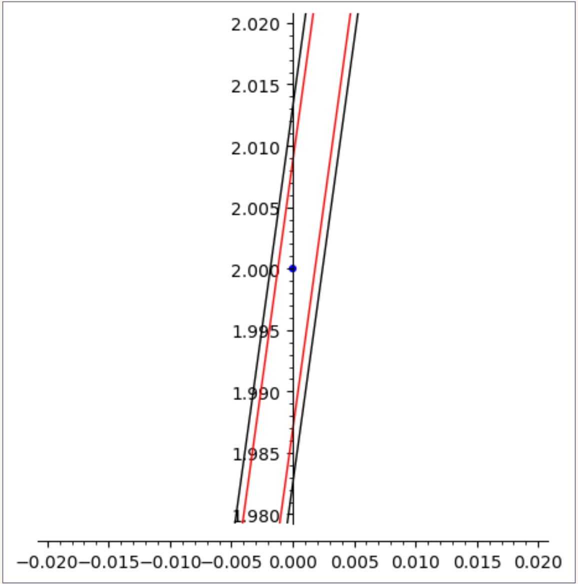
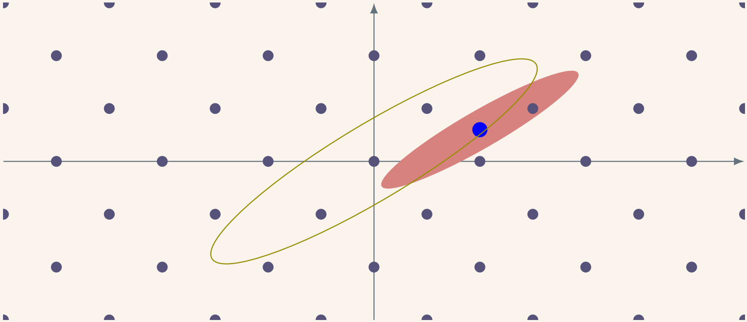
- **Lowner-John ellipsoid**



Sage plot. Distribution ellipse is outside figure, black is new ellipse

Ellipsoid Intersections



- Does the distribution ellipse (after alpha cuts) ∩ EBDD ellipse give a smaller ellipsoid?

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5. Ellipsoidal intersection

Acknowledgements

I'd like to thank Prof. Dachman-Soled (University of Maryland) and Hunter Kippen (University of Maryland) for their mentorship through this project, and Ms. Angelique Bosse for all her guidance and support.

Questions?